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Microwave Photodiodes: Sensitivity as a Function of Bias and Geometry

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ABSTRACT

A simple model of the microwave semiconductor photodiode has been examined theoretically with an emphasis on the dependence of sensitivity on bias, frequency, and geometrical factors. The noise equivalent power (*NEP*) of a small diode, a large diode, and an array of small diodes has been compared. According to this theory the large diode and the array of diodes with the same total active area have the same *NEP* at low frequencies, but the array is superior in high-frequency *NEP*. These conclusions are independent of diode bias. Curves of *NEP* vs frequency are presented for an array of nine diodes and for an array of 100 diodes.

PROBLEM STATUS

The work described in this report is a part of a more comprehensive and continuing project. This is a final report on this phase of the project.

AUTHORIZATION

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MICROWAVE PHOTODIODES: SENSITIVITY AS A FUNCTION OF BIAS AND GEOMETRY

INTRODUCTION

It is now widely appreciated that laser carriers open up the possibility of transmitting information at far faster rates than is possible with conventional microwave carriers. If a laser carrier system is to operate at high information rates, however, the light beam must be modulated at frequencies well into the microwave region, and there must be photodetectors capable of response in the microwave region. The name "microwave photodetector" has been given to such devices. Normally, the term "microwave photodiode" refers to a particular type of microwave photodetector in the form of a semiconductor *pn* junction.

One of the difficulties with the semiconductor microwave photodiode is that small junctions are required for good high frequency response, but large junctions are often needed to collect a large fraction of the light. One possibility is simply to build a larger junction and accept the loss in high frequency sensitivity. In principle, an array of small junctions would increase the light collected without affecting the frequency response. It was decided to examine the theory of microwave photodiodes to compare the properties of a small junction, a large junction, and an array of junctions. The most important parameter is the noise equivalent power NEP, which is defined as follows. Imagine a 100% amplitude-modulated light beam of average power $P_i = nh\nu$, where n is the photon flow in the entire cross section of the light beam (number/sec), h is Planck's constant, and ν is the light frequency. The beam is sine-wave modulated at a microwave radian frequency ω. The output of the photodiode will consist of a signal at frequency ω plus noise centered around ω, which is proportional to the bandwidth $d\omega$. For a given diode, biased at a given point, and for a given frequency and bandwidth, there will be one particular value of P_i which will produce a signal-to-noise ratio of 1 at the output of the photodetector. This value of P_i is the NEP. It is small, of course, for a good diode.

ANALYSIS

There are several treatments of the microwave photodiode in the literature. Some use models which are more complicated than needed for our purpose — the study of the NEP as a function of the diode geometry and bias. Johnson (1) uses a simple model, but we disagree with some of his results. It was decided, therefore, to make a fresh start, using Johnson's model.

Figure 1 is a generally accepted equivalent circuit for a microwave photodiode which is being excited by a beam of light of cross-sectional area A, which is much larger than the diode area a. The symbols used in Fig. 1 are defined as follows:

i_g = short-circuit current generated by the light beam,

i_j = shot noise current of the junction (assumed to be noninjecting at all biases considered),

G = *ac* junction conductance (bias dependent),

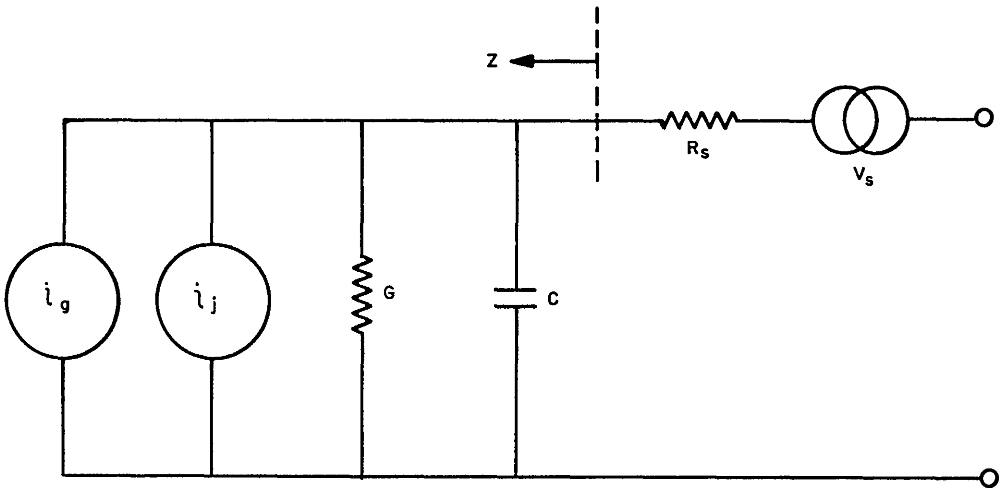

Fig. 1 - Equivalent circuit of microwave photodetector

C = junction capacitance (bias dependent),

R_s = ohmic resistance in series with the junction, and

v_s = thermal noise voltage from R_s.

We get Fig. 2 from Thévenin's theorem, in which all generators are voltage generators and $Z = 1/(G + j\omega C)$; $|Z|^2 = 1/(G^2 + w^2C^2)$. We can now calculate either the ratio of the available signal power to the available noise power or the ratio of the delivered signal power to the delivered noise power for an arbitrary load R_L. In either case, both R_L and $R_s + Z$ will cancel out, and we will get

$$\frac{S}{N} = \frac{|v_g|^2}{\overline{v_j^2} + \overline{v_s^2}} = \frac{|i_g|^2|Z|^2}{\overline{i_j^2}|Z|^2 + \overline{v_s^2}} = \frac{|i_g|^2}{\overline{i_j^2} + \overline{v_s^2}/|Z|^2}. \tag{1}$$

By definition, NEP is the input signal power required for a unity S/N. Therefore,

$$|i_g|^2 = \overline{i_j^2} + \overline{v_s^2}/|Z|^2 \tag{2}$$

Now, i_g is the short-circuit current produced by the incoming light and

$$i_g = Wn\eta q \frac{a}{A}, \tag{3}$$

where

$$W = \int e^{-\alpha x}\, dx\,,^*$$

*This integral is taken over the region in which carriers are collected. Roughly it is over the depletion region; thus, it is bias dependent.

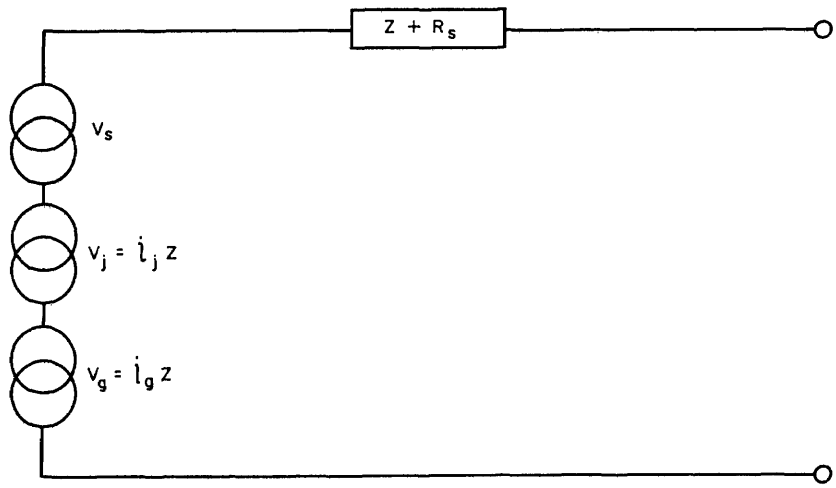

Fig. 2 - Thévenin's equivalent of Fig. 1

α is the absorption coefficient, n is the number of photons/sec in the light beam, η is the quantum efficiency, and q is the electronic charge. Since the total light power input in the beam is

$$P_i = nh\nu \ , \tag{4}$$

the short-circuit current i_g can be written as

$$i_g = \frac{P_i}{h\nu} \eta q \frac{a}{A} W = \mu P_i \ , \tag{5}$$

where

$$\mu = \frac{\eta q}{h\nu} \frac{a}{A} W \ .$$

Substituting Eq. 5 into Eq. 2, we obtain

$$\mu P_i = (\overline{i_j^2} + \overline{v_s^2}/|Z|^2)^{1/2} \ . \tag{6}$$

We define t_j and t_s by

$$\overline{i_j^2} = 4Kt_jT_0Gdf$$

and

$$\overline{v_s^2} = 4Kt_sT_0R_sdf \ ,$$

where t_j and t_s are the noise temperature ratios of the junction and of R_s.

Substituting the above definitions into Eq. 6, we obtain

$$\mu P_i = \left\{ 4KT_0 Gdf \left[t_i + t_s \frac{R_s}{G} (G^2 + \omega^2 C^2) \right] \right\}^{1/2} \tag{7}$$

It is customary to assume that $\omega^2 C^2 >> G^2$. Then,

$$P_i = NEP = \frac{1}{\mu} (4KT_0 Gdf)^{1/2} \left(t_j + t_s \frac{R_s C^2 \omega^2}{G} \right)^{1/2}$$

and

$$NEP = \frac{1}{\mu} \left[4KT_0 t_j Gdf (1 + t_s R_s \omega^2 C^2 / t_j G) \right]^{1/2} \tag{8}$$

One of the aims of this report is to consider the operation as a function of bias. The bias-dependent quantities are C, G, t_j, μ, and ω. We note that t_j and G appear only in the combination $t_j G$. The diode characteristic is assumed to be of the form*

$$I = I_0 [\exp (\beta V) - 1] ,$$

where I_0 is a constant, $\beta = q/(kT_A)$, and T_A is the ambient temperature. Therefore, $G = dI/dV = \beta I_0 \exp(\beta V) = G_0 \exp(\beta V)$. G_0 is thus the barrier conductance at zero bias.

It is easily shown that the noise temperature ratio for this same idealized diode (2) is given by

$$t_j = \frac{1}{2} \frac{\exp(\beta V) + 1}{\exp(\beta V)} t_s .$$

Note that this is approximately one-half t_s for forward bias, exactly t_s at zero bias, and becomes large at back bias. Thus,

$$G t_j = G_0 [\exp(\beta V) + 1] \; t_s .$$

We will consider only back bias. For convenience let us reverse the usual notation and assign positive numbers V to the back bias. Then the previous equation becomes

$$G t_j = G_0 [\exp(-\beta V) + 1] \; t_s . \tag{9}$$

The part in the brackets varies only between 1 and 2. Substituting Eq. 9 into Eq. 8, we get

*In some measurement schemes, one is forced to deal with a very low modulation index (on the order of 1%). This introduces an additional complication in that the light produces a dc current in the junction which can easily be larger than the saturation current of the diode. However, it would appear that any practical AM system would require modulation indices approaching 100%. Furthermore, in defining NEP, it does not seem that the diode should be penalized because of noise which is not really inherent in the diode. Therefore, in the present treatment it will be assumed that the modulation index is 100%, in which case (in the absence of carrier multiplication) the shot noise is due only to the electrical bias.

$$NEP = (1/\mu)\left(4KT_0\,df\,(G_0/2)\,[\exp(-\beta V) + 1]\,t_s\left\{1 + \frac{t_s R_s C^2 \omega^2}{(G_0/2)\,[\exp(-\beta V) + 1]}\right\}\right)^{1/2} \tag{10a}$$

$$= (Q^{1/2}/\mu)\,\{(G_0/2)\,[\exp(-\beta V) + 1]\,t_s\,(1 + \omega^2/\omega_c^2)\}^{1/2}\ , \tag{10b}$$

where

$$Q = 4KT_0\,df$$

and

$$\omega_c^2 = (G_0/2)\,[\exp(-\beta V) + 1]/(t_s R_s C^2)\ .$$

For zero bias, at room temperature

$$NEP = Q^{1/2}/\mu\,[G_0\,(1 + \omega^2/\omega_{co}^2)]^{1/2}\ , \tag{11}$$

where

$$\omega_{co}^2 = G_0/(t_s R_s C_0^2)\ .$$

This agrees with Johnson's result, although he fails to mention that his derivation holds only at zero bias.

At back bias of more than a fraction of a volt

$$NEP = (Q^{1/2}/\mu)\,[(G_0\,t_s/2)\,(1 + \omega^2/\omega_{cb}^2)]^{1/2}\ , \tag{12}$$

where

$$\omega_{cb}^2 = (G_0/2)/(t_s R_s C^2)\ .$$

Returning to Eq. 10a, it would be nice to have one equation in which the dependence on bias is explicit. There is an important class of diodes for which

$$C^2 = C_0^2 V_0/(V + V_0)\ ,$$

where C_0 and V_0 are the capacitance at zero bias and the built-in voltage, respectively. Substituting into Eq. 10a, we get an expression which is explicit in the bias dependence of the frequency response:

$$NEP = (Q^{1/2}/\mu)\left((G_0/2)\,[\exp(-\beta V) + 1]\,t_s\left\{1 + \frac{t_s R_s C_0^2 V_0\,\omega^2}{(G_0/2)\,[\exp(-\beta V) + 1]\,(V + V_0)}\right\}\right)^{1/2}\ , \tag{13}$$

which for appreciable back bias becomes

$$NEP = (Q^{1/2}/\mu)\ [(G_0\ t_s/2)\ \left(1 + \frac{t_s R_s C_0^2 V_0 \omega^2}{G_0 V/2}\right)^{1/2} \tag{14}$$

and for zero bias is

$$NEP = (Q^{1/2}/\mu)\ [G_0\ t_s \left(1 + \frac{t_s R_s C_0^2 \omega^2}{G_0}\right)^{1/2} \tag{15}$$

In these equations μ is still strongly bias dependent, since the volume which efficiently collects microwave hole-electron pairs is strongly bias dependent (Fig. 5, Ref. 1). As a practical matter, it appears that one should operate punched through (at least to within a microwave diffusion length)* to both contacts.

In comparing Eqs. 14 and 15, we see that since V_0/V can be considerably smaller than 1/2, the cutoff frequency can be higher for back bias than for zero bias.

SIZE EFFECTS

It is assumed that there will be practical applications in which it will be desirable (e.g., if one does not wish to use a lens system) to deal with beam areas which are considerable larger than the area of conventional junction microwave photodiodes. One obvious technique would be to concentrate on efforts (there may be serious technological difficulties, however) to construct very large area junctions. However, since the high-frequency properties of the diode will deteriorate with larger areas, it is not al all clear what the net result would be. Another proposal would be to construct an array of small diodes, thus collecting more light and preserving the high frequency response.

Consider then:

1. A single junction of area a.

2. A large junction of area Na.

3. N small junctions† of area a.

The parameters which will be affected are R_s, G, C, and μ. Equation 5 says that μ is directly proportional to the junction area. Since both C and G are associated with one-dimensional current flow, it is clear that they are both directly proportional to the area. In a well-designed diode, R_s will be due mainly to the current in the semiconductor, very close to the junction. This current will not be at right angles to the junction; i.e., it will not be a one-dimensional situation. It is well known that, for a circular contact, R_s is inversely proportional to the square root of the junction area. It is this single fact, namely that the square root of the junction area rather than the junction area determines R_s, which is the cause of the high-frequency degradation for large-area junctions.

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*The distance a carrier can diffuse in one-half of a microwave period.

†It is clear that for case 3 above there will of necessity be some dead space between junctions and that, therefore, the amount of real estate used on the semiconductor wafer will be larger than for B. In a practical device, some choice of spacing of the small junctions would have to be made, thus introducing at least one more parameter in the design of the device. In this report, we have avoided this problem by making the active areas equal in cases 2 and 3.

We rewrite Eq. 8 to emphasize the area-dependent parameters:

$$NEP = (M/\mu)\ [G(1 + \omega^2 \gamma R_s C^2/G)]^{1/2} , \tag{16}$$

where

$$M^2 = 4KT_0\, t_j\ df \text{ and } \gamma = t_s/t_j .$$

For case 1 simply assume that Eq. 16 refers to the junction area a.

For case 2

$$NEP_2 = \frac{1}{N} (M/\mu) \left[NG \left(1 + \frac{\omega^2 \gamma R_s N^2 C^2}{N^{1/2} NG}\right)\right]^{1/2}$$

and

$$NEP_2 = \frac{1}{N^{1/2}} (M/\mu) \left[G \left(1 + \frac{\omega^2 N^{1/2} \gamma R_s C^2}{G}\right)\right]^{1/2} .$$

Therefore, the cutoff frequency is reduced.

For case 3

$$NEP_3 = \frac{1}{N} (M/\mu) \left[NG \left(1 + \frac{\omega^2 \gamma R_s N^2 C^2}{N^2 G}\right)\right]^{1/2}$$

and

$$NEP_3 = \frac{1}{N^{1/2}} (M/\mu)\ [G(1 + \omega^2 \gamma R_s C^2/G)]^{1/2} .$$

Note that the cutoff frequency is unchanged.

It is useful to take the ratio

$$NEP_1/NEP_3 = N^{1/2} .$$

That is, the array scheme improves the sensitivity by $N^{1/2}$ at all frequencies (the cutoff frequency is unchanged).

Also,

$$NEP_1/NEP_2 = N^{1/2} \left(\frac{1 + \omega^2/\omega_c^2}{1 + N^{1/2} \omega^2/\omega_c^2}\right)^{1/2} .$$

At very low frequencies this approaches $N^{1/2}$ (i.e., it is equivalent to the array scheme). At very high frequencies it goes to $N^{1/4}$. It is easily seen, in fact, that the above ratio is larger than one for any frequency. Consider the square of the above ratio,

$$N \frac{1 + \omega^2/\omega_c^2}{1 + N^{1/2}\,\omega^2/\omega_c^2} > \frac{1 + N\omega^2/\omega_c^2}{1 + N^{1/2}\,\omega^2/\omega_c^2},$$

which is greater than one for any frequency if N is greater than one. Therefore, there is some improvement at all frequencies.

Note that these conclusions are true, independent of bias. In comparing a small junction with a large junction and with a small junction array, bias plays no role.

In Fig. 3, the expressions for NEP_1, NEP_2, and NEP_3 were used to calculate the relative NEP for a single junction, for a larger junction, and for a multijunction for $N = 9$ and also for $N = 100$. The plots are for a diode with a cutoff frequency f_c of 10 GHz, where $f_c^2 = G/(2\pi)^2 \gamma R_s C^2$.

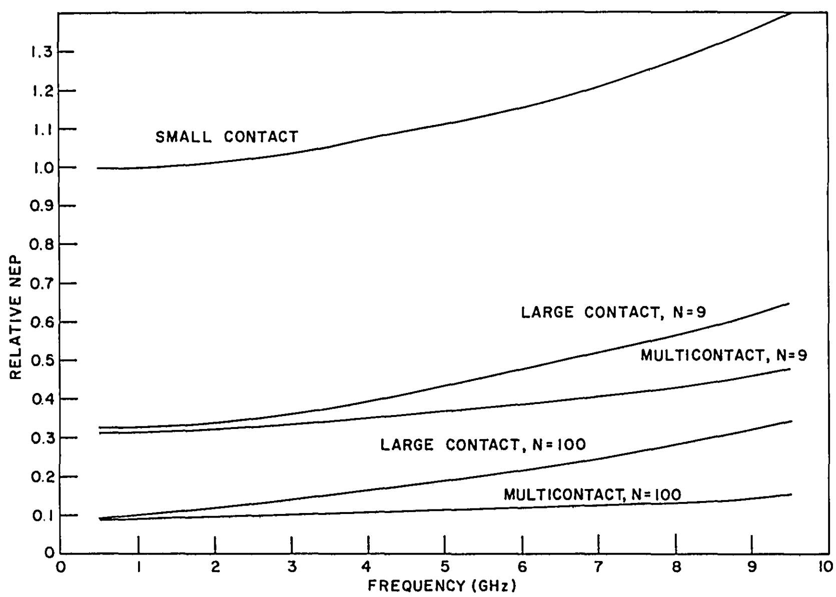

Fig. 3 - Comparison of the NEP for a small diode, a large diode, and a diode array

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This work was undertaken as a result of a proposal by Howard M. Day.

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2. Cowley, M.A., and Zettler, R.A., IEEE Trans. ED-15:761 (1968)

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